

# EvolSimple

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Date: 2003/01/30 14:31:41

## Abstract

This thorn illustrates a simple solver for the 3+1 equations, using the Cactus Einstein framework. It uses the “standard ADM” formulation of the evolution equations, and double leapfrog to implement the time integration. It assumes zero shift and no matter.

## 1 Physical System

The line element is

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij} dx^i dx^j, \quad (1)$$

where  $\alpha$  is the lapse and  $\gamma_{ij}$  the 3-metric. Defining  $n$  to be the normal to the slice, we have the extrinsic curvature  $K_{ij}$  given by

$$K_{ij} = \frac{1}{2} \mathcal{L}_n \gamma_{ij} \quad (2)$$

where  $\mathcal{L}$  is the Lie derivative.

The ADM equations then evolve the spatial three metric  $\gamma_{ij}$  and the extrinsic curvature  $K_{ij}$  using

$$\frac{d}{dt} \gamma_{ij} = -2\alpha K_{ij}, \quad (3)$$

$$\frac{d}{dt} K_{ij} = -D_i D_j \alpha + \alpha \left( K K_{ij} - 2K_{ik} K^k_j \right), \quad (4)$$

with

$$\frac{d}{dt} = \partial_t. \quad (5)$$

Here  $D_i$  is the covariant derivative associated with the three-dimensional metric  $\gamma_{ij}$ .

## 2 Using This Thorn

Set the parameter `ADMBase::evolution_method` to "simple".