Abstract

Initial Data for the 3D Scalar Wave Equation

1 Purpose

This thorn provides different initial data for the 3D scalar wave equation.

2 Spherically Symmetric Solutions

The general spherically symmetric solution can be written

$$\Psi(r,t) = \frac{1}{r}(f(r + t) + g(r - t))$$

where the functions $f$ and $g$ can be freely chosen.

Making the additional requirement of time symmetry at $t = 0$, forces

$$f(r) = g(r)$$

Thus if the solution at $t = 0$ is given by $\phi(r)$, the general solution will be

$$\Psi(r,t) = \frac{1}{2r}((r + t)\phi(r + t) + (r - t)\phi(r - t))$$

3 Gaussian

The gaussian solution is spherically symmetric about the origin of the Cartesian coordinate system, and is time symmetric. The initial profile is

$$\phi(r) = A \exp(-r^2/\sigma)$$

with the solution at the origin being

$$\Psi(r = 0, t) = \left(1 - 2\frac{t^2}{\sigma}\right)\exp(-t^2/\sigma)$$

The Gaussian solution is set with the parameters

- amplitude = $A$
- sigma = $\sigma$